

MTH 111, Math for the Architects , Final Exam

Ayman Badawi

QUESTION 1. (16 points) Find $y' = dy/dx$ do not simplify

(i) $y = 2\text{Sec}(4x + 3) + (1 + \cos(5x + 1))^4$

(ii) $y = [\ln(3x + 1)][e^{(3x+1)}]$

(iii) $y = \ln \left(\frac{(x^3+x+1)^3}{(e^x+4x-5)^2} \right)$

(iv) $y = e^{(\tan(4x)+3x-1)} + \ln(4x^2 + x - 1) + 2x - 11$

QUESTION 2. (6 points) Find the equation of the tangent line to the curve $(x + 1)\sin(y - 4) + ye^x + \cos(x) + 2y - 13 = 0$ at the point $(0, 4)$

QUESTION 3. (10 points) Sketch the graph of $f(x) = \cos(x) + \sin(x) + 3$ defined on $[-\pi, \pi]$ by considering the first derivative and the second derivative of $f(x)$. [Between 0 and π note that $\cos(x) = \sin(x)$ when $x = \pi/4$ and $\sin(x) = -\cos(x)$ when $x = 3\pi/4$]

QUESTION 4. (8 points) You want to construct a right triangle with area = $2m^2$ so that the length of the hypotenuse is minimum. What should be the length of the base, length of the height, and length of the hypotenuse?

QUESTION 5. (4 points) Rotate the graph $y = \sqrt{4 - x^2}$ (note that the graph is a semi-circle) around the x -axis where x is between $x = -2$ and $x = 2$. Find the volume of the object you generated.

QUESTION 6. (6 points) Find the area of the region bounded by $y = x^3 + 2x^2 - 3x$, x -axis, $x = 0$, and $x = 2$.

QUESTION 7. (6 points)

$$\text{Lim}_{x \rightarrow 0} \frac{e^x - x - 1}{\sin(x) - x^2 - x}$$

(ii) $\text{Lim}_{x \rightarrow 1} \frac{\sqrt{x} - x}{e^{(x-1)} - 1}$

QUESTION 8. (8 points) Given the ellipse $x^2 + 4y^2 - 4x + 4y^2 = 0$. Find the center, the foci, and the constant k . Sketch a rough graph of the given ellipse.

QUESTION 9. (8 points) Given $(-4, 3)$ is the vertex of a parabola that has the directrix $x = 1$. Find the standard equation of the parabola and find its focus, say F . Let Q be a point on the parabola. Find $|QF|$. Sketch a rough graph of the given parabola.

QUESTION 10. (5 points) given $f'(x) = 2x + \cos(x) + e^x$ and $f(0) = 4$. Find the function $f(x)$.

QUESTION 11. (6 points) Given $Q = (2, \frac{14}{3})$ lies on the graph of $f(x) = \frac{1}{3}x^3 + ax^2 + bx + c$ such that $f'(2) = 0$ but Q is neither a local minimum point nor a local maximum point. Find the values of the constants a, b, c .

QUESTION 12. (5 points) Find a point, say Q , on the graph $y = \sqrt{x}$ that is the nearest to the point $(4.5, 0)$

QUESTION 13. (12 points)

(i) $\int \frac{1}{x+\sqrt{x}} dx$ [Hint: you may want to factor $x + \sqrt{x}$ some how!!!]

(ii) $\int e^{2x}(3 + e^{2x})^7 dx$

(iii) $\int \frac{\sin(x)+1}{\cos^2(x)} dx$

Faculty information

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